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Lack of oscillations in Dual-Phase-Lagging heat conduction for a porous slab subject to imposed heat flux and temperature

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Abstract

This study shows that the physical conditions necessary for thermal waves to materialize in Dual-Phase-Lagging porous media conduction are not attainable in a porous slab subject to a combination of constant heat flux and temperature (Neumann and Dirichlet) boundary conditions. It is demonstrated that the approximate equivalence between Dual-Phase-Lagging (DuPhlag) heat conduction model and the Fourier heat conduction in porous media subject to Lack of Local Thermal Equilibrium (La Lotheq) that suggested the possibility of thermal oscillations and resonance reveals a condition that cannot be fulfilled because of physical constraints.

Keywords: Dual-Phase-Lagging; Hyperbolic heat conduction; Local thermal equilibrium; Porous media; Thermal waves

1. Introduction

This is a companion paper to Vadasz [1] as a complementary study aiming at demonstrating that oscillations are not possible in Dual-Phase-Lagging heat conduction in a porous slab subject to a combination of Dirichlet and Neumann boundary conditions. Repetition will therefore be kept to a minimum and presented only for the purpose of consistency and flow of presentation.

The system of governing equations for Fourier conduction in porous media subject to Lack of Local Thermal Equilibrium (La Lotheq) was showed by Tzou [2] to be approximately equivalent to the Dual-Phase-Lagging (DuPhlag) model of heat conduction. The latter can produce thermal waves in the form of oscillations. As a result the Dual-Phase-Lagging (DuPhlag) model can yield thermal resonance when periodically forced by a periodic heat source or a periodic boundary condition with a forcing frequency that is equal to one of the natural frequencies of the system. Tzou [2–4] presents applications of the DuPhlag model to a wide variety of fields from ultrafast (femtosecond) pulse-laser heating of metal films, phonon–electron interaction at nano and micro-scale heat transfer, temperature pulses in superfluid liquid helium, thermal lagging in amorphous materials, and thermal waves under rapidly propagating cracks.

Analytical solutions as well as analysis of the DuPhlag heat conduction were presented among others in excellent papers by Xu and Wang [5], Wang et al. [6], and Wang and Xu [7] and Antaki [8].

Applications of porous media heat transfer subject to Lack of Local Thermal Equilibrium (La Lotheq) were undertaken among others by Nield [9], Minkowycz et al. [10], Banu and Rees [11], Baytas and Pop [12],

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Nomenclature

$c_{p,\mathrm{f}}$, c_{s}	fluid and solid phase specific heat, respectively (dimensional)
c_n	dimensionless damping coefficient defined by Eq. (26)
$\hat{\boldsymbol{e}}_x$	unit vector in the x direction
$\hat{\boldsymbol{e}}_y$	unit vector in the y direction
$\hat{\boldsymbol{e}}_z$	unit vector in the z direction
Fo_q	heat flux related Fourier number, equals $\alpha_e\tau_q/L^2$
Fo_T	temperature gradient related Fourier number, equals $\alpha_e\tau_T/L^2$
N_β	Dual-Phase-Lagging bi-harmonic term dimensionless group, equals $\beta_e/\alpha_e L^2$
h	integral heat transfer coefficient for the heat conduction at the solid–fluid interface (dimensional)
k_{s}	effective thermal conductivity of the solid phase, equals $(1-\varphi)\tilde{k}_{\mathrm{s}}$ (dimensional)
$\tilde{k}_{\mathrm{s}}$	thermal conductivity of the solid phase (dimensional)
k_{f}	effective thermal conductivity of the fluid phase, equals $\varphi\tilde{k}_{\mathrm{f}}$ (dimensional)
$\tilde{k}_{\mathrm{f}}$	thermal conductivity of the fluid phase (dimensional)
L	the length of the porous slab (dimensional)
$\boldsymbol{q}$	heat flux vector (dimensional)
t_*	time (dimensional)
T	temperature (dimensional)
T_{C}	coldest wall temperature (dimensional)
x_*	horizontal co-ordinate (dimensional)
$\boldsymbol{x}$	position vector, equals $x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y + z\hat{\boldsymbol{e}}_z$

Greek symbols

α_e	effective thermal diffusivity, defined by Eq. (5) (dimensional)
β_e	effective property coefficient to the Dual-Phase-Lagging bi-harmonic term, defined in Eq. (5) (dimensional)
γ_{s}	solid phase effective heat capacity, equals $(1-\varphi)\rho_{\mathrm{s}}c_{\mathrm{s}}$ (dimensional)
γ_{f}	fluid phase effective heat capacity, equals $\varphi\rho_{\mathrm{f}}c_{p,\mathrm{f}}$ (dimensional)
θ	dimensionless temperature, equals $(T-T_{\mathrm{C}})/(T_{\mathrm{H}}-T_{\mathrm{C}})$
φ	porosity
ρ_{s}	solid phase density
ρ_{f}	fluid phase density
τ_q	time lag associated with the heat flux, defined by Eq. (5) (dimensional)
τ_T	time lag associated with the temperature gradient defined by Eq. (5) (dimensional)
ω_n	dimensionless natural thermal frequency defined by Eq. (26)

Subscripts

*	corresponding to dimensional values of the independent variables, except for cases where there is no ambiguity, as listed in this nomenclature
s	related to the solid phase
f	related to the fluid phase

Kim and Jang [13], Rees [14], Alazmi and Vafai [15], and Nield, Kuzentsov and Xiong [16]. While the significance of practically obtaining the same temperature solution for each phase in a porous medium subject to a Lack of Local Thermal Equilibrium (La Lotheq) is discussed by Vadasz [17] identifying conditions for which the traditional formulation of the La Lotheq model is not adequate, the conditions used in the present paper are not identical to those identified by Vadasz [17]. Other examples of conditions that are not affected by the conclusions of the present paper are problems of convection subject to La Lotheq such as those presented by Spiga and Morini [18], Kuznetsov [19], Amiri and Vafai [20] and Kuznetsov [21].

The present paper deals with Fourier heat conduction in a porous medium subject to La Lotheq. It aims at demonstrating that the condition required for oscillatory solutions, $\tau_T/\tau_q < 1$, is not physically attainable in a porous slab conduction subject to a combination of an imposed constant heat flux (Neumann) and constant temperature (Dirichlet) boundary conditions. While the results of the present paper may provide a useful guidance among others to pulsed laser processing of nanofilms (e.g. [22]), the problem presented here is essentially distinct and deals with the application to porous media. There are major distinctions as well as similarities between the two. The similarities are linked to the two-phase coupled equations used to represent the "absorption of photon energy by electrons and the heating of the lattice through electron–phonon coupling" [23]. The distinctions are mainly in the small scale of the ultra fast heating of metals or thin films leading to a legitimate use of a non-Fourier constitutive model to represent the relationship between the heat flux and the temperature gradient, such as the application of the Dual-Phase-Lagging for each phase (Hays-Stang and Haji-Sheikh [22]). In porous media due to the typical macroscopic scale of both phases the latter is not applicable. In the present paper Fourier Law was employed for the heat flux mechanism at each phase.

Nevertheless the coupling between the phases in terms of the heat conduction at the solid-fluid interface leads to a formulation that is approximately equivalent to Dual-Phase-Lagging. The latter is a result of the analysis and not an imposed constitutive relationship.

2. Problem formulation

2.1. Governing equations for lack of local thermal equilibrium

The heat conduction equations for the two phases that compose an isotropic and homogeneous porous medium are obtained as phase averages over a Representative Elementary Volume (REV) following *Fourier's Law* in the form

$$\gamma_s \frac{\partial T_s}{\partial t_*} = k_s \nabla_*^2 T_s - h(T_s - T_f) \tag{1}$$

$$\gamma_f \frac{\partial T_f}{\partial t_*} = k_f \nabla_*^2 T_f + h(T_s - T_f) \tag{2}$$

where $\gamma_s = (1-\varphi)\rho_s c_s$ and $\gamma_f = \varphi\rho_f c_{p,f}$ are the solid phase and fluid phase effective heat capacities, respectively, φ is the porosity, k_s and k_f are the effective thermal conductivities of the solid and fluid phases, respectively, and h represents an integral heat transfer coefficient for the heat conduction at the solid-fluid interface within an REV, assumed to be independent of time and anticipated to depend on the thermal conductivities of both phases, on the porosity, on the heat transfer surface area and on the tortuousity of the interface between the solid and fluid phases [24,25]. In the case of fluid flow the value of h will depend also on local Reynolds and Prandtl numbers of the fluid as presented by Alazmi and Vafai [15].

When the Local Thermal Equilibrium assumption is not valid, conditions appropriate for the case when the temperature difference between the two phases is not small, the two equations (1) and (2) are to be solved simultaneously. The diffusion terms in these equations are a result of replacing the $-\nabla_*\cdot\boldsymbol{q}_s$ and $-\nabla_*\cdot\boldsymbol{q}_f$ terms by using Fourier's Law in the form $\boldsymbol{q}_s = -k_s\nabla_* T_s$ and $\boldsymbol{q}_f = -k_f\nabla_* T_f$ to yield the Laplacian terms. The coupling between the two equations can be resolved as presented by Vadasz [1,17] leading to

$$\left[\left(\gamma_s \frac{\partial}{\partial t_*} - k_s\nabla_*^2 + h\right)\left(\gamma_f \frac{\partial}{\partial t_*} - k_f\nabla_*^2 + h\right) - h^2\right] T_i = 0 \quad \forall i = s, f \tag{3}$$

where the index i can take the values s representing the solid phase or f standing for the fluid phase. The explicit form of Eq. (3) is obtained after dividing it by $h(\gamma_s + \gamma_f)$ in the form

$$\tau_q \frac{\partial^2 T_i}{\partial t_*^2} + \frac{\partial T_i}{\partial t_*} = \alpha_e\left[\nabla_*^2 T_i + \tau_T \nabla_*^2\left(\frac{\partial T_i}{\partial t_*}\right) - \beta_e \nabla_*^4 T_i\right] \quad \forall i = s, f \tag{4}$$

where the following notation was used

$$\tau_q = \frac{\gamma_s\gamma_f}{h(\gamma_s + \gamma_f)}; \quad \alpha_e = \frac{(k_s + k_f)}{(\gamma_s + \gamma_f)};$$
$$\tau_T = \frac{(\gamma_s k_f + \gamma_f k_s)}{h(k_s + k_f)}; \quad \beta_e = \frac{k_s k_f}{h(k_s + k_f)} \tag{5}$$

In Eqs. (4) and (5) τ_q and τ_T are the heat flux and temperature related time lags linked to the Dual-Phase-Lagging (DuPhlag) to be discussed below, while α_e is the effective thermal diffusivity of the porous medium. It may be observed from Eq. (5) that there is a dual effect of the heat capacities on the effective parameters of the uncoupled system in the sense that the heat flux time lag τ_q is affected by the heat capacities of the solid and fluid phases as thermal capacitors connected in series following the relationship $1/\gamma_e^s = 1/\gamma_s + 1/\gamma_f = (\gamma_s + \gamma_f)/\gamma_s\gamma_f$, while the effective thermal diffusivity is affected by the heat capacities of the solid and fluid phases as thermal capacitors connected in parallel following the relationship $\gamma_e^p = (\gamma_s + \gamma_f)$. In addition the parameter β_e can be presented as the ratio between the effective thermal conductivity due to the thermal resistances of the solid and fluid phases connected in series and the heat transfer coefficient h , in the form $\beta_e = k_e/h$, where $k_e = k_s k_f/(k_s + k_f)$ and the thermal resistance of each phase is defined as $1/k_i\ \forall i = s, f$.

2.2. Governing equations for Dual-Phase-Lagging heat conduction

The Dual-Phase-Lagging model applied to porous media conduction was introduced by Tzou [2] and its solution was presented among others by Xu and Wang [5]. In the Dual-Phase-Lagging model the following formulation is suggested to replace the classical Fourier Law [2]

$$\boldsymbol{q}_i(\boldsymbol{x}_*, t_* + \tau_{qi}) = -k_i\nabla_* T_i(\boldsymbol{x}_*, t_* + \tau_{Ti}) \quad \forall i = s, f \tag{6}$$

where the relationship between the heat flux and temperature gradient is not instantaneous but rather affected by two time lags, a heat flux lag τ_{qi}, and a temperature gradient time lag, τ_{Ti}. By expanding Eq. (6) in a Taylor series in time and truncating the series at the first order approximation it yields (see Vadasz [1], Tzou [2] for details) to order $O(\tau_{qi})$ and $O(\tau_{Ti})$

$$\boldsymbol{q}_i + \tau_{qi}\frac{\boldsymbol{q}_i}{\partial t_*} = -k_i\left[\nabla_* T_i + \tau_{Ti}\frac{\partial(\nabla_* T_i)}{\partial, t_*}\right] \quad \forall i = s, f \tag{7}$$

This Dual-Phase-Lagging formulation is applied to the thermal conduction energy equation

$$\gamma_i \frac{\partial T_i}{\partial t_*} + \nabla_* \cdot \boldsymbol{q}_i = 0 \quad \forall i = s, f \tag{8}$$

by replacing the fluid-solid interface heat transfer term with the Dual-Phase-Lagging formulation. Applying now the $(\nabla_* \cdot)$ operator on Eq. (7), substituting $\nabla_* \cdot \boldsymbol{q}_i = -\gamma_i\, \partial T_i/\partial t_*$ from Eq. (8) and dividing the resulting equation by γ_i yields one equation for the temperature of each phase due to Dual-Phase-Lagging in the form

$$\tau_{qi} \frac{\partial^2 T_i}{\partial t_*^2} + \frac{\partial T_i}{\partial t_*} = \alpha_i \left[\nabla_*^2 T_i + \tau_{Ti} \nabla_*^2 \left(\frac{\partial T_i}{\partial t_*} \right) \right] \quad \forall i = s, f \tag{9}$$

Eq. (9) is the conduction Dual-Phase-Lagging equation for each phase of a porous medium. Comparing Eq. (9) with the uncoupled equations (4) obtained from applying the Fourier Law to each phase, while including the fluid-solid interface heat transfer term, shows that they are equivalent provided the bi-harmonic term in Eq.(4) is negligibly small, i.e. if $\beta_e \sim 0$, and provided the following equivalency of parameters is enforced

$$\tau_{qi} = \tau_q = \frac{\gamma_s \gamma_f}{h(\gamma_s + \gamma_f)}; \quad \alpha_i = \alpha_e = \frac{(k_s + k_f)}{(\gamma_s + \gamma_f)};$$
$$\alpha_e \tau_{Ti} = \frac{(\gamma_s k_f + \gamma_f k_s)}{h(\gamma_s + \gamma_f)} \quad \forall i = s, f \tag{10}$$

Therefore the consequent definition of the temperature gradient lag, τ_{Ti}, that is consistent with Dual-Phase-Lagging is

$$\tau_{Ti} = \tau_T = \frac{(\gamma_s k_f + \gamma_f k_s)}{h(k_s + k_f)} \quad \forall i = s, f \tag{11}$$

In the present paper Fourier law was invoked at all stages and the approximate Dual-Phase-Lagging was obtained as a result of the heat transfer interaction between the phases, not imposed as a constitutive relationship instead of Fourier law.

A direct property of these parameters is by evaluating the ratio τ_T/τ_q by using Eq. (5), which leads to the following result

$$\frac{\tau_T}{\tau_q} = 1 + \frac{\gamma_s^2 k_f + \gamma_f^2 k_s}{\gamma_s \gamma_f (k_s + k_f)} > 1 \tag{12}$$

Since the combination of positive valued properties in the second term of Eq. (12) is always positive, the time lags ratio is always greater than 1, i.e. $\tau_T/\tau_q > 1$. The latter conclusion that is based on a physical argument and it is accurately derived has a profound impact on the following results. It applies generally to Fourier heat conduction in porous media subject to La Lotheq and is not restricted to any specific geometry nor boundary conditions. Note that while each one of the time lags τ_T and τ_q depend on the interface heat transfer coefficient h as observed in Eqs. (5), (10) and (11), their ratio τ_T/τ_q in Eq. (12) is independent of this coefficient making its evaluation simpler as it depends on the effective properties of each phase and is independent of the interaction between the phases.

3. Analytical solution

The analysis of the Dual-Phase-Lagging model for porous media conduction is undertaken for a particular solution of Eq. (9) corresponding to the one dimensional heat conduction in a porous slab of length L as presented in Fig. 1. The first major distinction between the present problem pertaining to a combination of constant temperature and constant heat flux imposed on the boundaries and the problem presented by Vadasz [1] that applied to constant temperatures imposed on both boundaries is that the present problem does not allow for identical temperature solutions for both solid and fluid phases as long as the effective thermal conductivities of the phases are distinct [17]. Transforming Eq. (4) into a dimensionless form by using L to scale the independent length variable x_*, i.e. $x = x_*/L$, by using L^2/α_e to scale the time, i.e. $t = \alpha_e t_*/L^2$, and introducing the dimensionless temperature, θ_i,

$$\theta_i = \frac{(T_i - T_C)k_f}{q_L L} \quad \forall i = s, f \tag{13}$$

where T_C is the cold wall imposed temperature and the heat flux at $x_* = L$ is represented in the form $(q)_{x_*=L} = -q_L = \text{const.} < 0$, defining the value of $q_L > 0$, and leading to

$$Fo_q \frac{\partial^2 \theta_i}{\partial t^2} + \frac{\partial \theta_i}{\partial t} = \frac{\partial^2 \theta_i}{\partial x^2} + Fo_T \frac{\partial^3 \theta_i}{\partial t \partial x^2} - N_\beta \frac{\partial^4 \theta_i}{\partial x^4}$$
$$\forall i = s, f \tag{14}$$

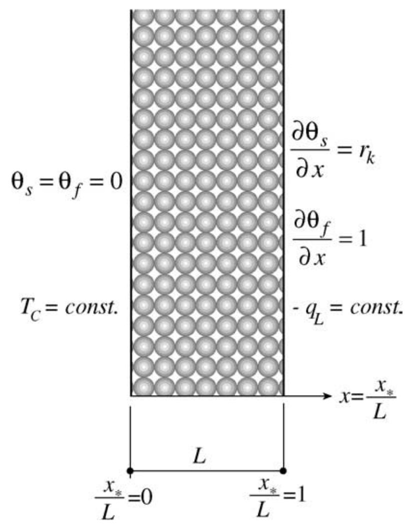

Fig. 1. A fluid saturated porous slab subject to a combination of constant temperature and constant heat flux conditions at the walls.

where two Fourier numbers, Fo_q, Fo_T, and one additional dimensionless group are defined as

$$Fo_q = \frac{\alpha_e \tau_q}{L^2}; \quad Fo_T = \frac{\alpha_e \tau_T}{L^2}; \quad N_\beta = \frac{\beta_e}{L^2} \tag{15}$$

The consistency between the Dual-Phase-Lagging and the two-phase porous media equations require that $N_\beta \ll 1$. Then, the bi-harmonic term in Eq. (14) is neglected according to Tzou [2] leading to

$$Fo_q \frac{\partial^2 \theta_i}{\partial t^2} + \frac{\partial \theta_i}{\partial t} = \frac{\partial^2 \theta_i}{\partial x^2} + Fo_T \frac{\partial^3 \theta_i}{\partial t \partial x^2} \quad \forall i = s, f \tag{16}$$

and their corresponding boundary and initial conditions are

$$x = 0 : \theta_i = 0 \quad \forall i = s, f \tag{17}$$

$$x = 1 : \left(\frac{\partial \theta_s}{\partial x}\right)_{x=1} = 1; \quad \left(\frac{\partial \theta_f}{\partial x}\right)_{x=1} = r_k \tag{18}$$

where $r_k = k_f/k_s$ is the effective thermal conductivity ratio.

$$t = 0 : \theta_i = \theta_0 = \text{const. and } \dot{\theta}_i = \dot{\theta}_0 = \text{const.} \quad \forall i = s, f \tag{19}$$

The solution to Eq. (16) is separated into steady state $\theta_{i,(ss)}$ and transient $\theta_{i,(tr)}$ parts in the form $\theta = \theta_{i,(ss)} + \theta_{i,(tr)} \forall i = s,f$. The steady state is represented by the linear solutions $\theta_{f,(ss)} = x$ and $\theta_{s,(ss)} = r_k x$, which satisfy the boundary conditions Eqs. (17) and (18). The transient solutions $\theta_{f,(tr)}$ and $\theta_{s,(tr)}$ have to fulfil the equation

$$Fo_q \frac{\partial^2 \theta_{i,(tr)}}{\partial t^2} + \frac{\partial \theta_{i,(tr)}}{\partial t} = \frac{\partial^2 \theta_{i,(tr)}}{\partial x^2} + Fo_T \frac{\partial^3 \theta_{i,(tr)}}{\partial t \partial x^2} \quad \forall i = s, f \tag{20}$$

and the following boundary and initial conditions

$$x = 0 : \theta_{i,(tr)} = 0 \quad \forall i = s, f \tag{21}$$

$$x = 1 : \left(\frac{\partial \theta_{i,(tr)}}{\partial x}\right)_{x=1} = 0 \quad \forall i = s, f \tag{22}$$

$$t = 0 : \theta_{f,(tr)} = (\theta_0 - x), \quad \theta_{s,(tr)} = (\theta_0 - r_k x) \text{ and } \dot{\theta}_{i,(tr)} = \dot{\theta}_0 \quad \forall i = s, f \tag{23}$$

The solution is obtained by separation of variables in the form of two equations for each phase $\theta_{i,(tr)} = \phi_n(t) u_n(x)$, $\forall i = s,f$, where the functions $\phi_n(t)$ and $u_n(x)$ are identical for both phases because of the identical boundary conditions (21) and (22). These equations are

$$\frac{d^2 \phi_n}{dt^2} + c_n \frac{d\phi_n}{dt} + \omega_n^2 \phi_n = 0 \tag{24}$$

$$\frac{d^2 u_n}{dx^2} + \kappa_n u_n = 0 \tag{25}$$

The solution of Eq. (25) subject to the homogeneous boundary conditions $u_n = 0$ at $x = 0$ and $(du_n/dx)_{x=1} = 0$ at $x = 1$ is $u_n = a_n \sin(\kappa_n x)$ and the resulting eigenvalues are $\kappa_n = \pi/2 + n\pi = (2n + 1)\pi/2 \quad \forall n = 0, 1, 2, 3, \ldots$ The coefficients c_n and ω_n^2 in Eq. (24) are defined in the form

$$c_n = Fo_q^{-1}(1 + \kappa_n^2 Fo_T); \quad \omega_n^2 = Fo_q^{-1}\kappa_n^2 = (4Fo_q)^{-1}(2n+1)^2\pi^2 \tag{26}$$

Eq. (24) represents a linear damped oscillator. Its eigenvalues are

$$\lambda_{1n} = -\frac{c_n}{2}\left[1 + \sqrt{1 - 4\frac{\omega_n^2}{c_n^2}}\right] \tag{27}$$

$$\lambda_{2n} = -\frac{c_n}{2}\left[1 - \sqrt{1 - 4\frac{\omega_n^2}{c_n^2}}\right] \tag{28}$$

The solution for ϕ_n *is overdamped* if for some values of n the condition $4\omega_n^2 < c_n^2$ is satisfied, leading to

$$\theta_{tr,n} = (A_{1n} e^{\lambda_{1n} t} + A_{2n} e^{\lambda_{2n} t}) \sin(\kappa_n x), \tag{29}$$

it is critically damped if for some values of $n = n_{cr}$ the condition $4\omega_{n_{cr}}^2 = c_{n_{cr}}^2$ is satisfied, i.e. $\lambda_{1n} = \lambda_{2n} = \lambda_{n_{cr}} = -c_{n_{cr}}/2$ leading to

$$\theta_{tr,n_{cr}} = (A_{1n_{cr}} e^{\lambda_{ncr} t} + A_{2n_{cr}} t e^{\lambda_{ncr} t}) \sin(\kappa_{n_{cr}} x), \tag{30}$$

and *it is underdamped* if for some values of n the condition $4\omega_n^2 > c_n^2$ is satisfied, i.e. $\lambda_{1n} = \lambda_r - i\lambda_i$ and $\lambda_{1n} = \lambda_r + i\lambda_i$, where $\lambda_r = -c_n/2$ and $\lambda_i = \sqrt{4\omega_n^2 - c_n^2}/2$, leading to decaying *thermal waves* in the form

$$\theta_{tr,n} = e^{-c_n t}\{A_{1n}[\cos(\lambda_i t - \kappa_n x) - \cos(\lambda_i t + \kappa_n x)] - A_{2n}[\sin(\lambda_i t - \kappa_n x) - \sin(\lambda_i t + \kappa_n x)]\} \tag{31}$$

4. Impossibility of oscillations and lack of resonance

The condition for an underdamped solution and its associated oscillations is further explored to obtain explicit criteria in terms of the primitive parameters of the original system. By using the definitions from Eq. (26) it produces the condition for an underdamped (oscillatory) solution in the form

$$\frac{c_n^2}{4\omega_n^2} = \frac{[4 + Fo_T(2n+1)^2\pi^2]^2}{16 Fo_q (2n+1)^2 \pi^2} < 1 \tag{32}$$

An analysis of inequality (32) presented in Appendix A produces the following necessary and sufficient condition for the underdamped solution to materialize

$$\frac{Fo_T}{Fo_q} = \frac{\tau_T}{\tau_q} < 1 \tag{33}$$

a condition that is identical to the one obtained when Dirichlet boundary conditions were applied on both walls [1]. However Eq. (12) shows that based on physical arguments the time lag ratio τ_T/τ_q is always greater than one, i.e. $\tau_T/\tau_q > 1$. Therefore, underdamped (oscillatory) solutions, which require according to Eq. (33) that $\tau_T/\tau_q < 1$, are being ruled out. Similarly, since the condition for critically damped solutions is $Fo_T/Fo_q = \tau_T/\tau_q = 1$, but in reality this ratio is greater than 1, i.e. $\tau_T/\tau_q > 1$, critically damped solutions are ruled out as well. One can therefore conclude that underdamped and critically damped solutions are not possible, and therefore oscillations cannot occur in the Dual-Phase-Lagging application to porous media conduction subject to the specified geometry and boundary conditions. Resonance could have been possible due to a forced periodic source or alternatively due to periodic boundary conditions at a forcing frequency that is identical to one of the natural frequencies of the system. However the lack of possibility for underdamped solutions prevent resonance from occurring in the Dual-Phase-Lagging application to porous media conduction subject to the specified geometry and boundary conditions.

5. Conclusions

The approximate equivalence between the Dual-Phase-Lagging (DuPhlag) heat conduction model and the Fourier heat conduction in porous media subject to Lack of Local Thermal Equilibrium (La Lotheq) lead to the expectation that thermal waves and resonance are possible. It was demonstrated that the conditions necessary for such thermal oscillations and possibly resonance to materialize are not physically attainable in a porous slab subject to a combination of Dirichlet and Neumann boundary conditions.

Appendix A

The condition for underdamped (oscillatory) solutions to materialize was presented in Eq. (32) in the form

$$\frac{[4 + Fo_T(2n+1)^2\pi^2]^2}{16Fo_q(2n+1)^2\pi^2} < 1 \tag{A.1}$$

By introducing the notation $m = 2n + 1$ $\forall n = 0, 1, 2, 3, \ldots$, or $\forall m = 1, 3, 5, 7, \ldots$ one can expand the inequality (A.1) to produce the following inequality which applies to the values of m

$$y \equiv m^4 + bm^2 + c < 0 \tag{A.2}$$

where

$$b = \frac{8(Fo_T - 2Fo_q)}{\pi^2 Fo_T^2}; \quad c = \frac{16}{\pi^4 Fo_T^2} > 0 \tag{A.3}$$

By treating m^2 as a continuous variable, the function $y(m^2) = m^4 + bm^2 + c$ represents a parabola which has a minimum at $m^2 = (-b/2)$. For obtaining real and positive values of m the roots, m^2, of the equation $y \equiv m^4 + bm^2 + c = 0$ have to be real and positive. The plot of $y(m^2)$ as a function of m^2 is presented in Fig. 2 identifying the cases where $c > 0$, while the cases when $c < 0$ are not presented as they are not applicable here according to Eq. (A.3). The two typical curves presented in Fig. 2 correspond to $b < 0$ ($Fo_T/Fo_q < 2$) and $b > 0$ ($Fo_T/Fo_q > 2$). For $b > 0$ ($Fo_T/Fo_q > 2$) the negative part of the curve corresponds to negative values of m^2 and therefore cannot accommodate real values of m. For $b < 0$ ($Fo_T/Fo_q < 2$) an underdamped solution is in principle possible provided the roots of the quadratic equation $y \equiv m^4 + bm^2 + c = 0$ are real. The latter implies that the following roots $m_{1,2}^2 = -b[1 \pm \sqrt{1 - 4c/b^2}]/2$ have to be real for obtaining two real and positive values of m^2 as presented in Fig. 2. For the latter to occur the discriminant $(b^2 - 4c)$ must be positive. By substituting the parameters from Eq. (A.3) into this condition, it yields

$$\frac{64}{\pi^4 Fo_T^2}\left[\frac{(Fo_T - 2Fo_q)^2}{Fo_T^2} - 1\right] > 0 \tag{A.4}$$

The only difference between this inequality and the one obtained in the case of Dirichlet boundary conditions for both walls is the value of the coefficient in front of the brackets. Further derivation of the square brackets in Eq. (A.4) accounting for positive values of $Fo_q > 0$ and $Fo_T > 0$ leads to

$$\frac{Fo_T}{Fo_q} = \frac{\tau_T}{\tau_q} < 1 \tag{A.5}$$

Inequality (A.5) represents a necessary and sufficient condition for underdamped solutions to materialize.

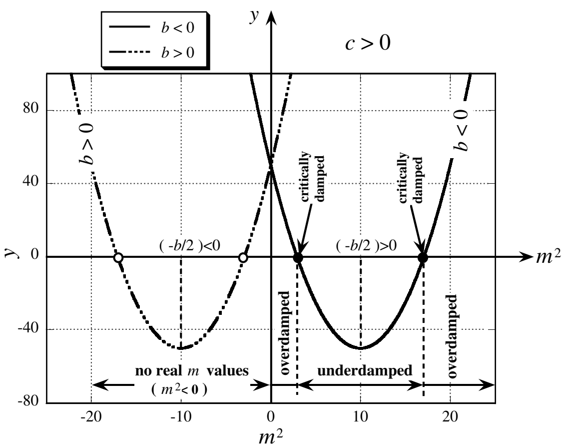

Fig. 2. Graphical representation of the conditions for underdamped, critically damped and overdamped solutions in terms of the function $y(m^2) = m^4 + bm^2 + c$, for $c > 0$.

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